Protecting Yao from Malicious Attacks

Mike Rosulek

Oregon State University

crypt@b-it 2018

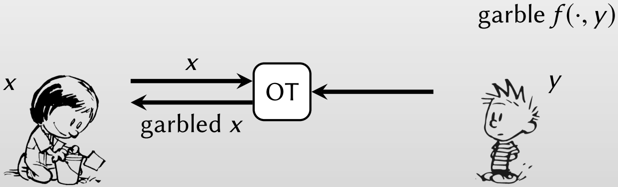
Yao's Protocol Recap

garble $f(\cdot, y)$

x

y

Yao's Protocol Recap



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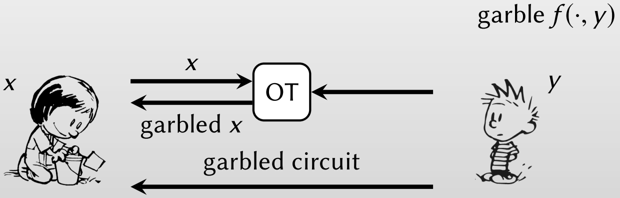

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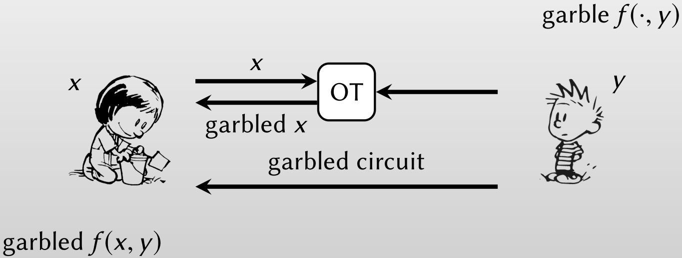

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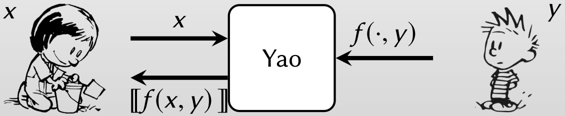

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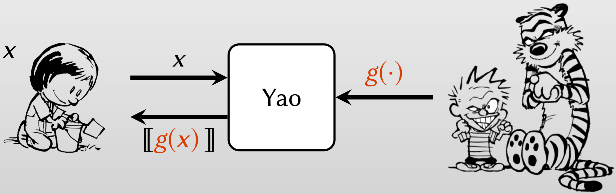

- Full security against malicious receiver
- Malicious sender can construct bad garbled circuit

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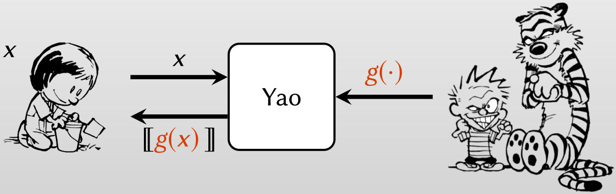


- Full security against malicious receiver
- Malicious sender can construct bad garbled circuit
  - (essentially the **only** thing that can go wrong with Yao)

Roadmap

1 **Cut-and-choose:**

- Concepts & mechanisms: reducing replication factor
- Security pitfalls & challenges

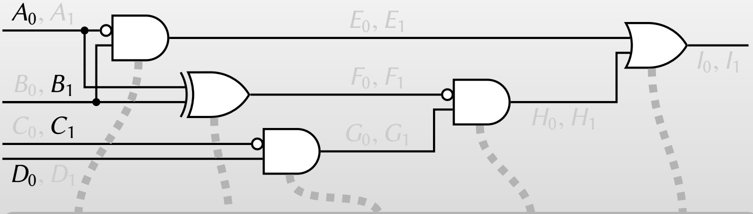
2 **Dual execution:** security minus 1 bit of leakage

3 **Batch setting:** economies of scale for repeated computations

Essence of cut-and-choose

How can you be sure that a garbled circuit was generated correctly?

Opening a garbled circuit



$\mathbb{E}_{A_0,B_0}(E_0)$	$\mathbb{E}_{A_0,B_0}(F_0)$	$\mathbb{E}_{C_0,D_0}(G_0)$	$\mathbb{E}_{F_0,G_0}(H_0)$	$\mathbb{E}_{E_0,H_0}(I_0)$
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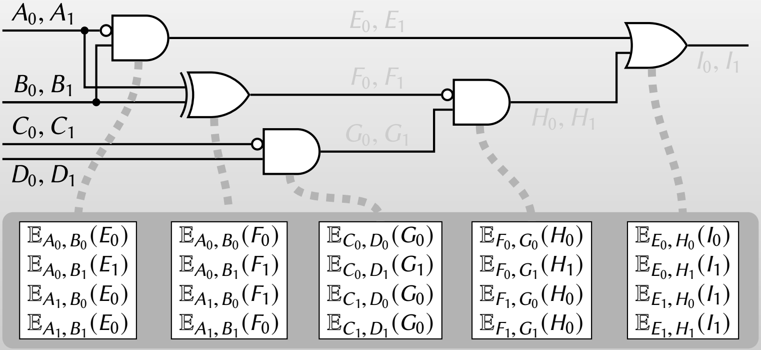

Seeing **all input labels** $\Rightarrow$ can check correctness of garbled gates

- (Better yet, give a seed to PRG that determines all input labels)

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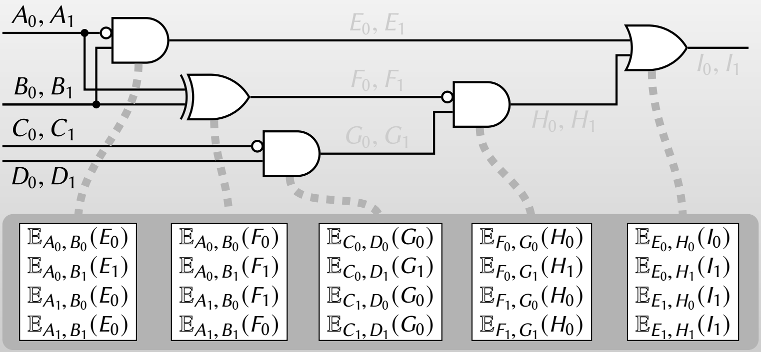
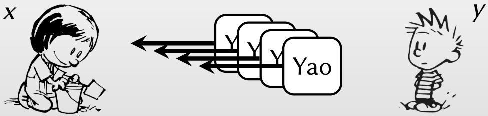


Seeing **all input labels** $\Rightarrow$ can check correctness of garbled gates

- (Better yet, give a seed to PRG that determines all input labels)
- This circuit no longer provides any privacy to computation!
- Can **open/check** a garbled circuit or use it for **evaluation**, not both!

Essence of cut-and-choose



Cut-and-choose approach:

1. Prepare for several **independent** instances of Yao's protocol

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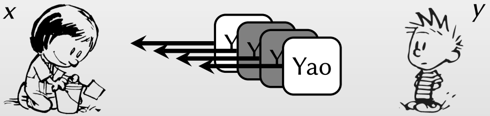

Cut-and-choose approach:

1. Prepare for several **independent** instances of Yao's protocol
2. **Open/check** some *random* subset of the garbled circuits
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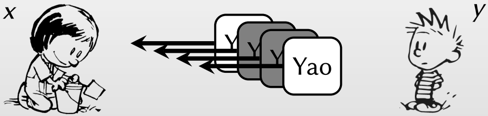

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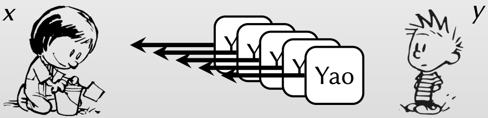
Questions:

- How many instances are needed? (replication factor) How many should be opened?
- How to actually do this without introducing new security flaws?

All-but-one [AumannLindell07]



Garble n copies

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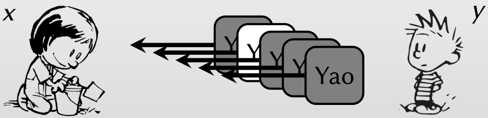

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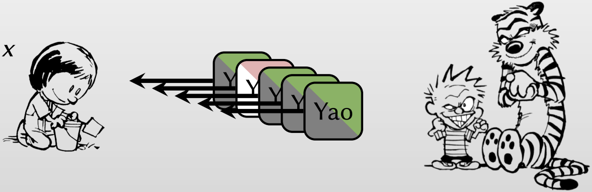

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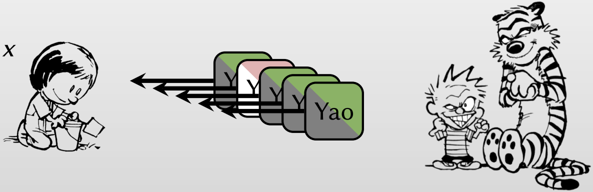

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$\Leftrightarrow$ Adv exactly predicts cut-choose challenge

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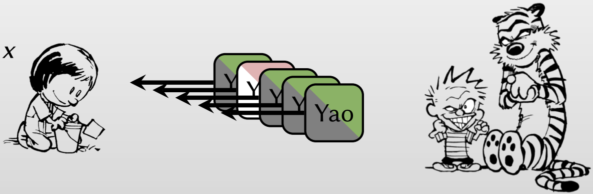

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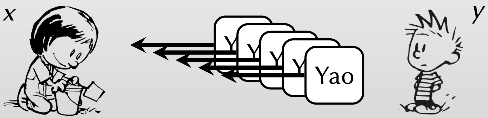
$\Leftrightarrow$ Adv exactly predicts cut-choose challenge

Adversary can win with probability $1/n$ (too high!)

Majority cut [LindellPinkas07]



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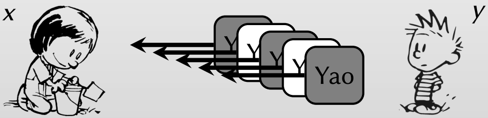

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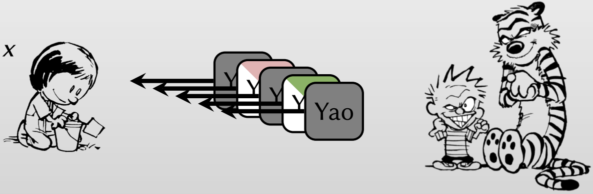

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Questions:

- Evaluate *several circuits* ⇒ what if some of them disagree?
- How many circuits? How many to open?

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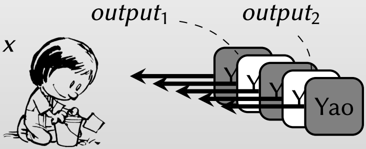

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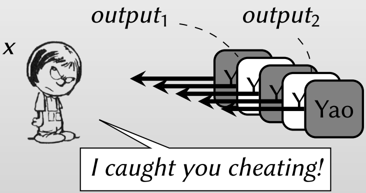

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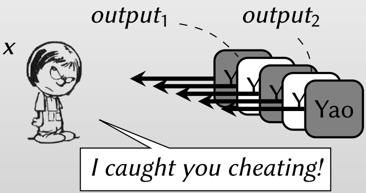

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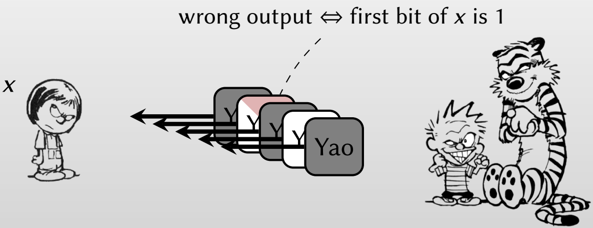

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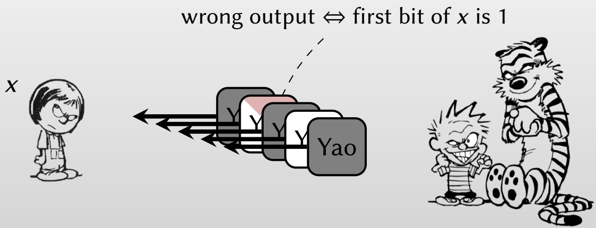

Suppose evaluated circuits disagree

- Garbler **must** be cheating ⇒ Evaluator should abort!

THIS IS INSECURE!

- Ability to *detect* cheating can depend on private input!
- Need another way to deal with disagreeing outputs!

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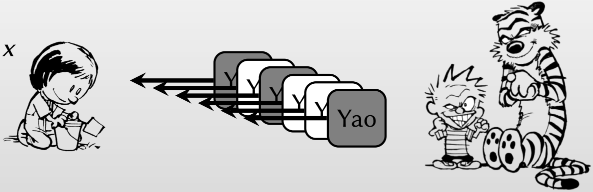

Idea: Accept the **majority** output of evaluated circuits.

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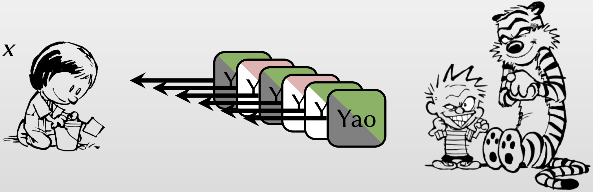

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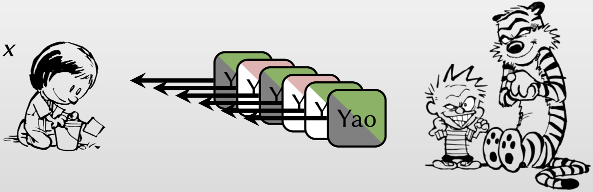

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[ShelatShen11]: To ensure $\Pr[\text{Adv wins}] < 2^{-s}$:

- Generate $\sim 3.12\, s$ circuits (replication factor)
- Open random subset of =60% of circuits
- For $s = 40$: generate 125 circuits and check 75

Majority cut **pitfalls**

*Even with **correct garbled circuits**,*
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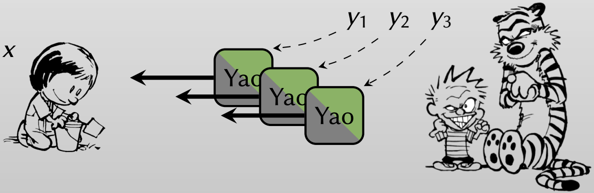

(**either party** could use inconsistent inputs!)

Evaluator input consistency

How to enforce **input consistency** for evaluator?

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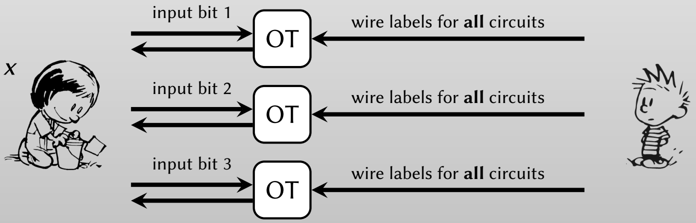
- **Easy:** use **one OT** for all evaluation circuits!



Garbler input consistency

How to enforce **input consistency** for garbler?

Idea: [ShelatShen13] compute the function $(x, y) \mapsto f(x, y) \| H(y)$

- Evaluator checks that $H(y)$ same for majority of circuits
- H should be **collision-resistant**
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Can arrange for y to be **committed** before H is chosen

- Can use simple 2-universal function H
- Example: $H(y)$ = multiplication by random (public) 0/1-matrix
  - $\Rightarrow$ computation of H free using Free-XOR garbling

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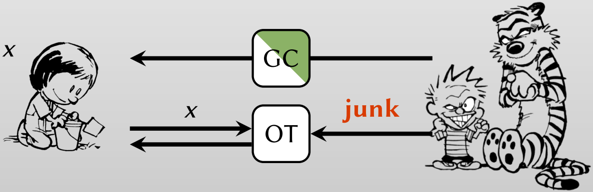

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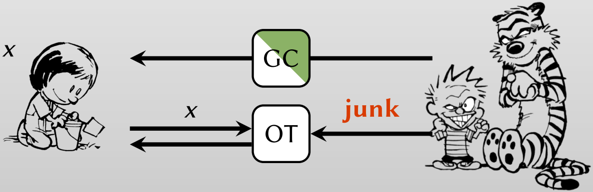

Selective failure attack: Garbler sends bad **input wire labels**

. . . conditioned on receiver's OT choice bits (her *private input!*)

- E.g.: junk wire label ⇔ first bit of x is 1

Selective failure prevention

How to avoid **selective failure** attack?

Idea: [LindellPinkas07,ShelatShen13] Make OT choice bits less sensitive

- Evaluate the function $\big((x_1, \ldots, x_k), y\big) \mapsto f(x_1 \oplus \cdots \oplus x_k, y)$
- Each input bit is secret-shared into k OT choice bits $\Rightarrow$ k-wise independence!

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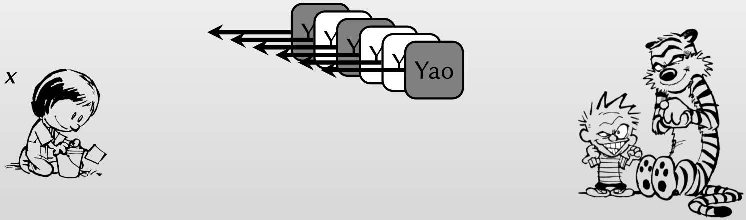
Analysis:

- Garbler "poisons" $< k$ OTs $\Rightarrow$ evaluator failure probability independent of x
- Garbler "poisons" $\geq k$ OTs $\Rightarrow$ evaluator failure probability $\geq 1 - 2^{-k}$

Cheating punishment [Lindell13]



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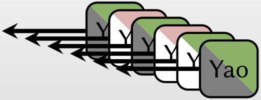

Question: Can we get security if **only one** evaluated circuit is good?

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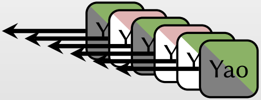
x



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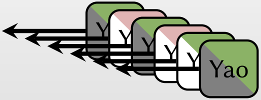

x

compute $f(x, y)$

p

y

if p is proof of cheating, then output garbler's input (else output $\perp$)

malicious secure computation

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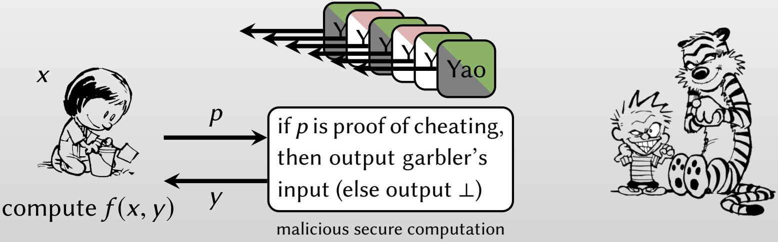
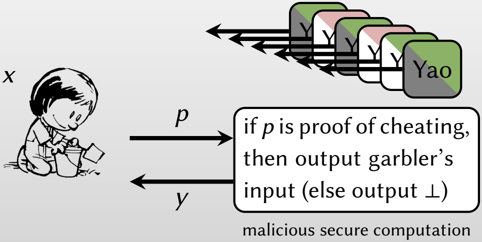
- However, evaluator cannot reveal whether she has such proof!
- Let her **privately** exchange cheating proof for garbler's input!

Cheating punishment: **details**



- Auxiliary secure computation uses majority-cut-and-choose
- Auxiliary computation depends only on **input length** of f
- **Many** many many optimizations to make aux computation small
- Must ensure **same input** y to both main & aux computations
- Evaluator can learn $f(x, y)$ in two ways, but can't reveal which!

Cheating punishment: **analysis**



With **just one** good evaluation circuit:

Case 1: All evaluation circuits agree on output

Case 2: Evaluation circuits disagree on output

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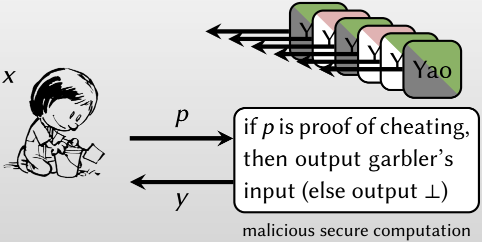

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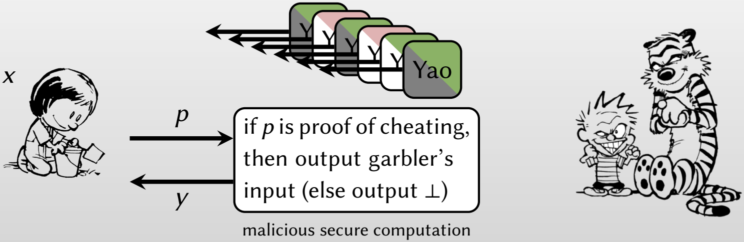

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$\Rightarrow$ evaluator gets proof of cheating $\Rightarrow$ evaluator gets correct $f(x, y)$

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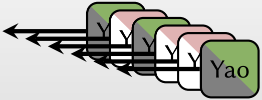

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← y

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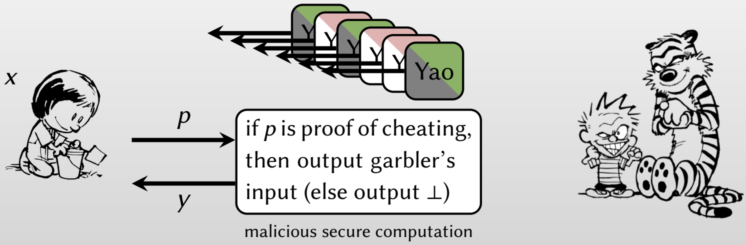

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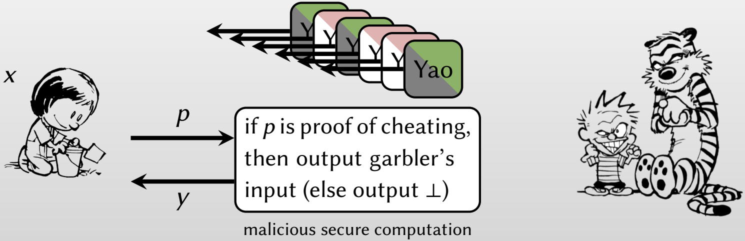

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$\Leftrightarrow$ Adv exactly predicts cut-choose challenge

Suppose each circuit is checked with independent probability 1/2

- With **only s circuits**, $Pr[\text{Adv wins}] \leq 2^{-s}$ (vs. $> 3s$ circuits)

Roadmap

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- Concepts & mechanisms: reducing replication factor
- Security pitfalls & challenges

2 **Dual execution:** security minus 1 bit of leakage

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dual execution protocol [MohasselFranklin06]

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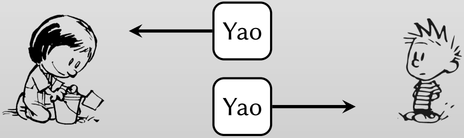
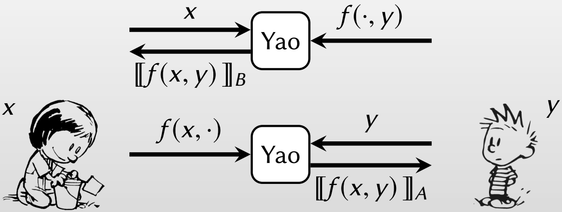
Yao's protocol is secure against malicious receiver



⇒ *run it in both directions!*

dual execution protocol [MohasselFranklin06]



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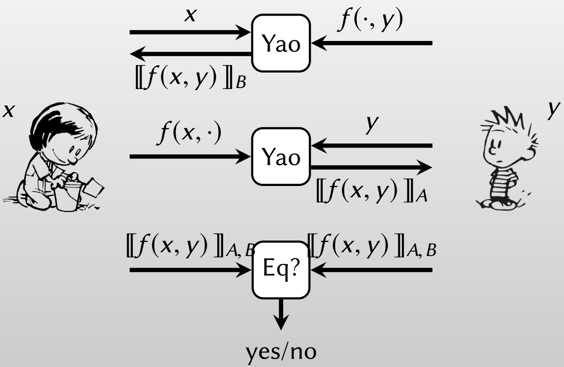

- Define a **common** garbled encoding: $[\![z]\!]_{A,B} \stackrel{\text{def}}{=} [\![z]\!]_A \oplus [\![z]\!]_B$
- Malicious Bob can't predict $[\![z]\!]_{A,B}$ for for $z \neq f(x,y)$ (authenticity)

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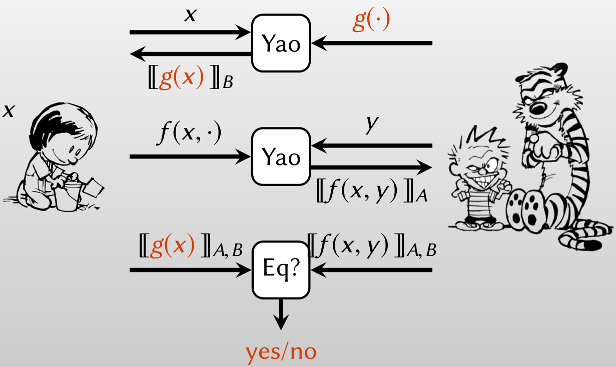

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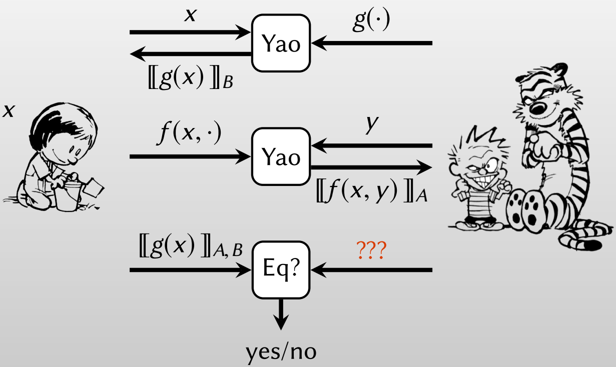

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- Malicious Bob can't make Alice accept incorrect output!

reducing leakage [KolesnikovMohasselRivaRosulek15]

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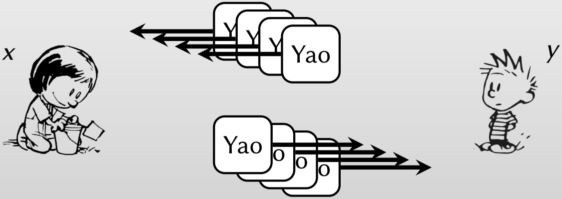
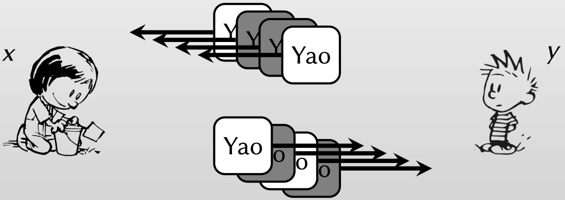


Main idea:

- Run s copies of Yao's protocol in each direction

reducing leakage [KolesnikovMohasselRivaRosulek15]



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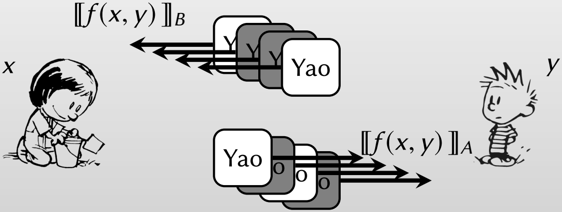

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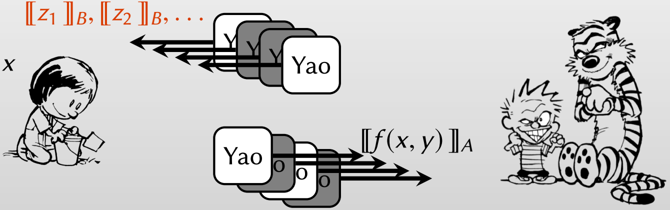

Main idea:

- Run *s* copies of Yao's protocol in each direction
- Cut and choose: check each garbled circuit with probability 1/2.
- Garbled circuits in same direction have same output encoding
- What to do when Alice gets disagreeing outputs?

reconciliation technique

$[\![z^*]\!]_B$

$[\![z^*]\!]_A$

- Honest parties can compute common $[\![z^*]\!]_{A,B} \stackrel{\text{def}}{=} [\![z^*]\!]_B \oplus [\![z^*]\!]_A$

reconciliation technique

$[\![z_1]\!]_B, [\![z_2]\!]_B, \ldots$

$S_A = \left\{ [\![z_i]\!]_{A,B} \right\}_i$

$[\![z^*]\!]_A$

- Honest parties can compute common $[\![z^*]\!]_{A,B} \overset{\text{def}}{=} [\![z^*]\!]_B \oplus [\![z^*]\!]_A$
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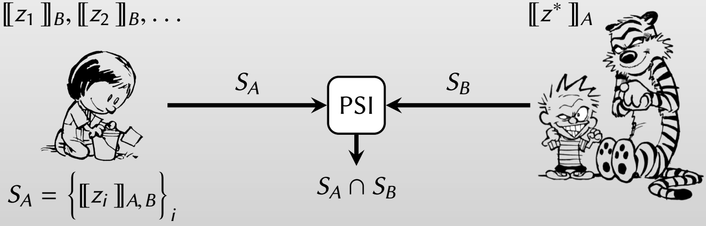

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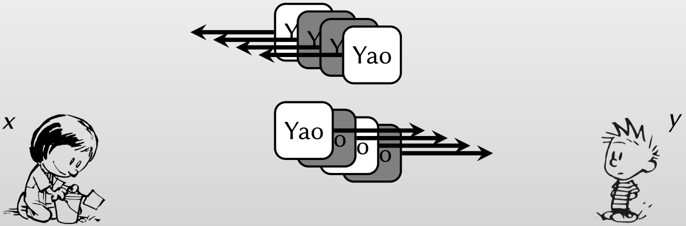
- If disagreeing outputs, compute **set of candidates**
- Do **private set intersection** on the sets!
  - $\Rightarrow$ PSI output identifies the "correct" z_i

protocol summary



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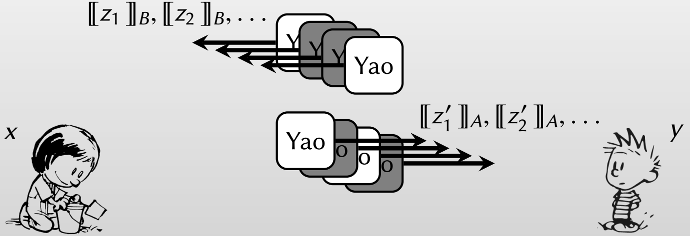
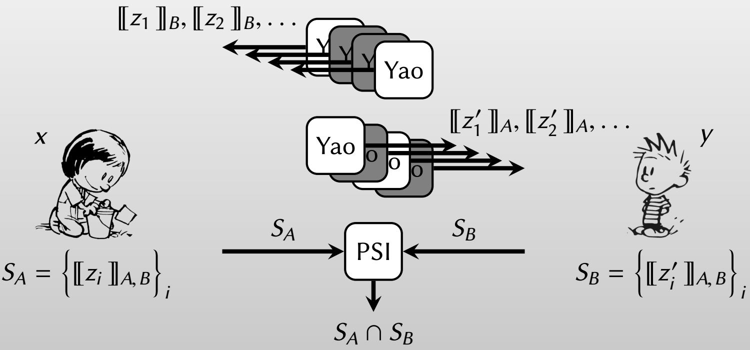


$$S_A = \left\{ [\![z_i]\!]_{A,B} \right\}_i \qquad S_B = \left\{ [\![z'_i]\!]_{A,B} \right\}_i$$

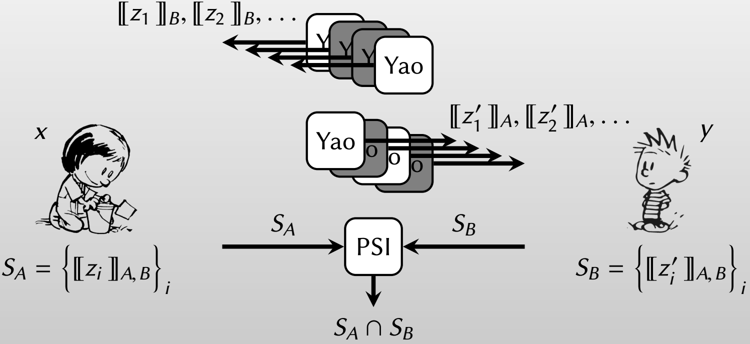
- s instances of Yao in each direction, check random subset
- Compute set of reconciliation values

protocol summary

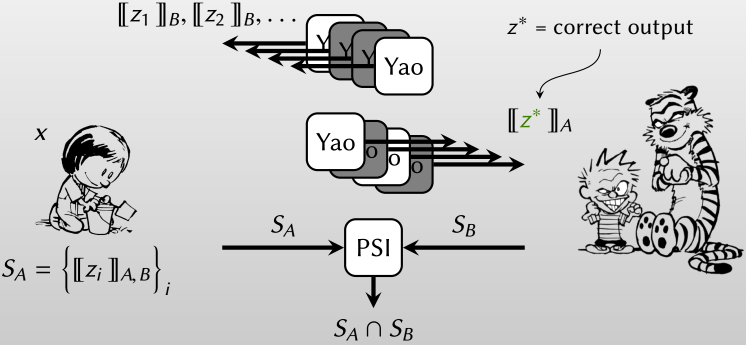


- s instances of Yao in each direction, check random subset
- Compute set of reconciliation values
- Private set intersection to identify correct output

protocol analysis



protocol analysis: corrupt Bob



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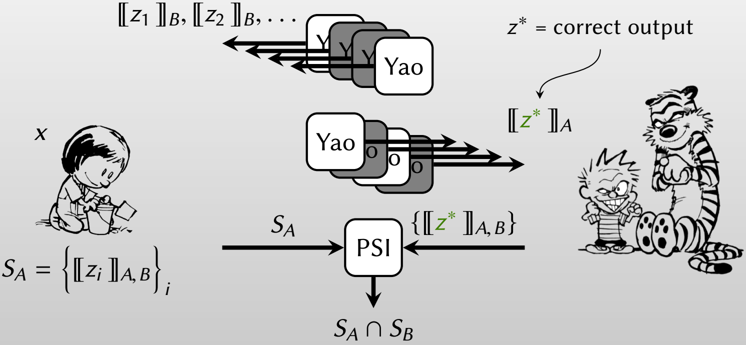

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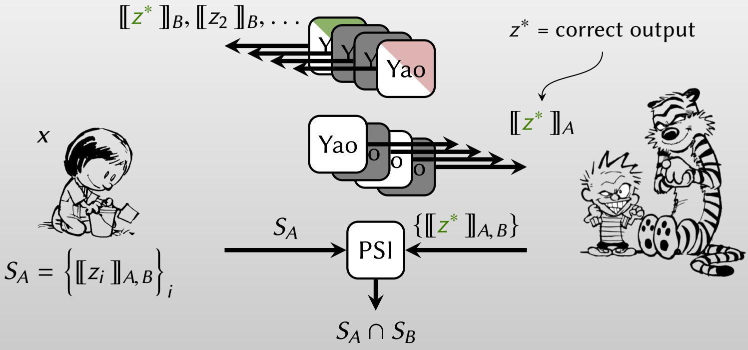

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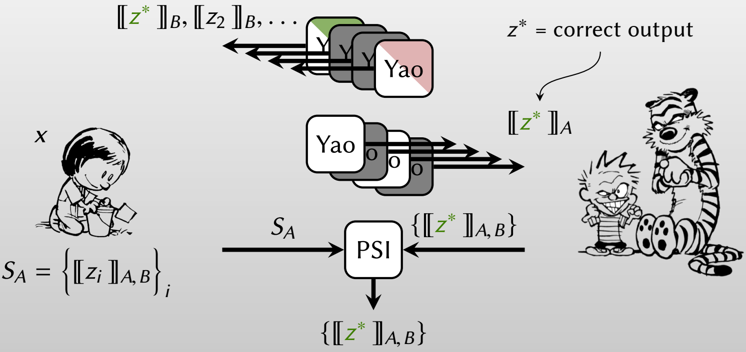

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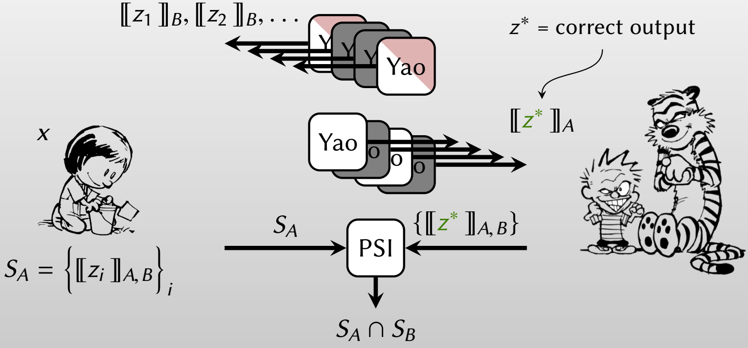

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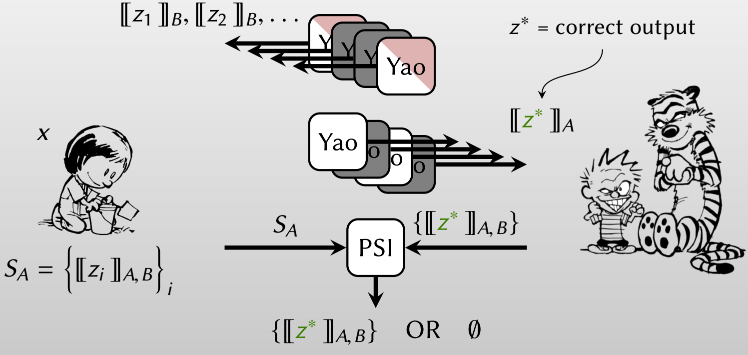

- Bob's only "useful" PSI input is $[\![z^*]\!]_{A,B}$
- **Just one** good evaluation circuit ⇒ PSI output leaks nothing!
- **All** evaluation circuits bad ⇒ PSI output leaks just 1 bit

“dual-ex+PSI” summary

s garbled circuits in each direction (can be done simultaneously)

Adversary cannot violate output correctness

Adversary learns a single bit **with probability** 2^{-s} — only when:

- All opened circuits are correct
- All evaluated circuits are incorrect

Example: only **10 circuits** for 0.1% chance of single-bit leakage

- all other security properties hold with overwhelming probability

Roadmap

1 **Cut-and-choose:**

- Concepts & mechanisms: reducing replication factor
- Security pitfalls & challenges

2 **Dual execution:** security minus 1 bit of leakage

3 **Batch setting:** economies of scale for repeated computations

online/offline setting

Want to do 2PC of same circuit N times?

[HuangKatzKolesnikovKumaresanMalozemoff14,LindellRiva14]

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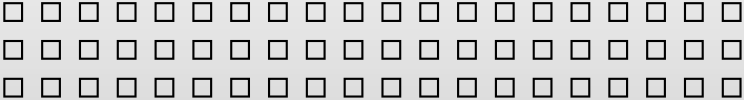

generate a lot of garbled circuits

online/offline setting

Want to do 2PC of same circuit N times?

[HuangKatzKolesnikovKumaresanMalozemoff14,LindellRiva14]

open and check some fraction of them

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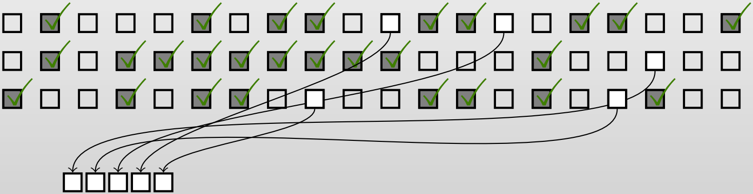

pick a random “bucket” of available circuits and evaluate them

online/offline setting

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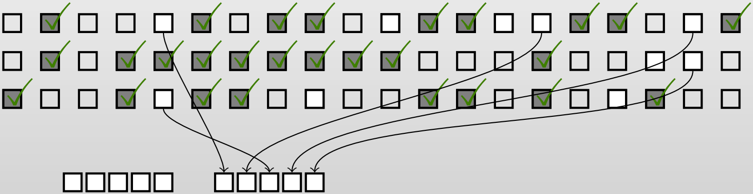

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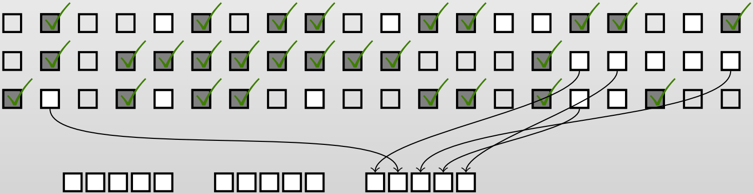

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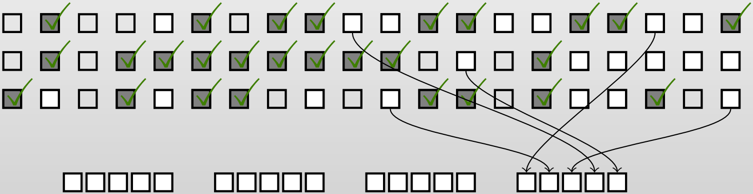

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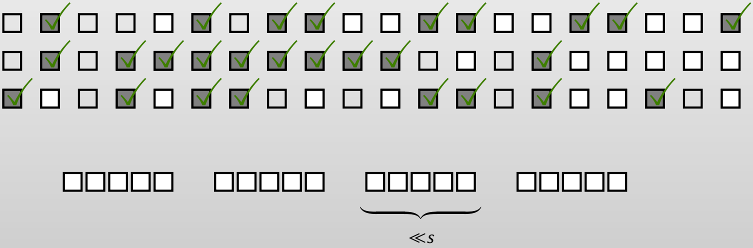

Adversary **wins** ⇔ { **all** opened circuits are good
some bucket has **all**/**maj** bad circuits

- for security $1/2^s$, need $2 + O(s/\log N)$ circuits per execution
- example: $N = 1024$, $s = 40$ $\implies$ **only** 4 circuits per execution

Cut-and-choose Perspective

Big Idea: Generate many garbled circuits; check some, evaluate others

- Traditional approach (majority evaulation): **125 circuits**
- Cheating punishment technique: **40 circuits**
- Willing to tolerate Pr[leak 1 bit] = 0.001: **10 circuits** (each direction)
- Willing to tolerate 1 bit of leakage: **2 circuits** (1 in each direction)
- Evaluating same circuit many times: **3 or 4 circuits** per evaluation

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Other approaches:

- **LEGO**: [NielsenOrlandi09,FJNNO13,FJNT15] cut-and-choose on **individual gates**, not circuits
  - Replication factor $2 + O(s/\log N)$ but now $N =$ # gates
  - Extra costs needed to connect gates together
- **DUPLO**: [KolesnikovNielsenRosulekTrieuTrifiletti17] cut-and-choose on **medium-size components** (between single gate and entire circuit)
- **Pool**: [ZhuHuangCassel17] maintain large fixed-size collection of garbled circuits to support **unlimited** number of evaluations