Proceedings of IDETC/CIE 2005
ASME 2005 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference
September 24-28, 2005, Long Beach, California, USA

DETC2005-84394

Modeling and Imaging Mechanical Chaos

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Abstract

The word "chaotic system" [Peitgen92] describes a system whose outputs are very sensitive to its initial conditions. Because of their inherent complex nature, chaotic systems are difficult to visualize and understand. This paper describes the visualization of a mechanical chaotic system – a magnetic pendulum. The program uses dynamics modeling and imaging, so that a user can experiment with different configurations and then visualize how that configuration responds to all input conditions. The result shows interesting patterns and insights into the mechanical system itself. This same technique would be applicable to visualizing many other chaotic systems.

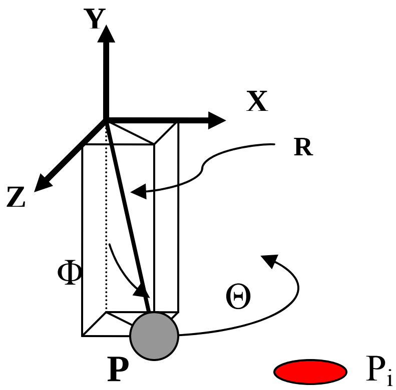
Introduction

> "The greatest problem that computers are confronted with when dealing with chaos is the extreme sensitivity of an iterator." [Peitgen92]

This photo shows a common device seen in science museums and on executive desks. It consists of a pendulum with a magnetic bob and (in this case) three magnets on the base. The user experiments with this by setting the pendulum in motion. Once swinging, it is immediately obvious just how interesting and complex this motion is. The bob alternately is attracted and then released by all three magnets. Eventually damping takes over and the pendulum settles down to being attached to a single magnet. It is also immediately obvious that this is chaotic motion. Because the motion is so complex, the final resting position of the pendulum is very sensitive to the initial conditions given by the user. We became interested in the nature of the motion and in determining and visualizing just how chaotic the motion is.

The Simulation

The following figure shows a diagram of the mechanical system. It has two degrees of freedom, Φ and Θ. Φ is how much the pendulum has swung away from the vertical. This is the angle that would be used to analyze a simple one-degree-of-freedom pendulum. The angle Θ is how much the pendulum revolves around in a horizontal plane.



P is the position of the pendulum at the current time. L is the length of the pendulum from the pivot point to the bob. The P_i are the positions of the magnets and the F_i are the strengths of the magnets. The total torque on the pendulum is a combination of the magnet forces and gravity acting on the bob as follows:

$$\bar{\tau}_T = \sum \bar{P} \times \frac{F_i * (\bar{P}_i - \bar{P})}{\left\| \bar{P}_i - \bar{P} \right\|^3} - Lmg\hat{j}$$

The resulting torque is a vector quantity because it has x, y, and z components. It is composed of two components, a Φ component that is perpendicular to the swinging motion and a Θ component that is perpendicular to the revolving motion. These two torque components drive the system, giving acceleration equations of:

$$\ddot{\Theta} = \frac{\tau_\Theta - (c_\Theta \dot{\Theta})}{m(L \sin \Phi)^2}$$

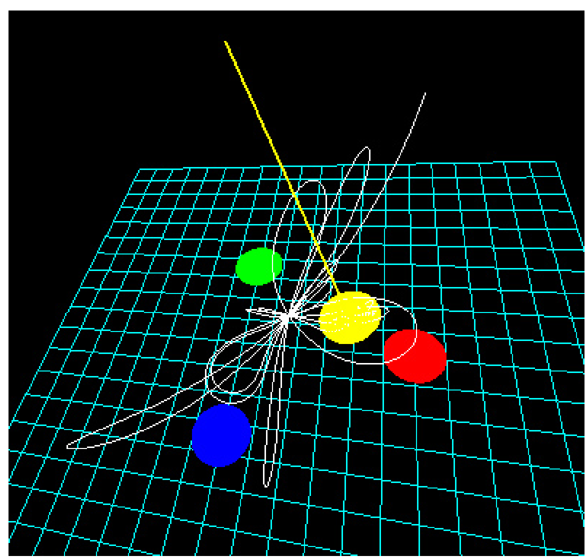
$$\ddot{\Phi} = \frac{\tau_\Phi - (c_\Phi \dot{\Phi})}{mL^2}$$

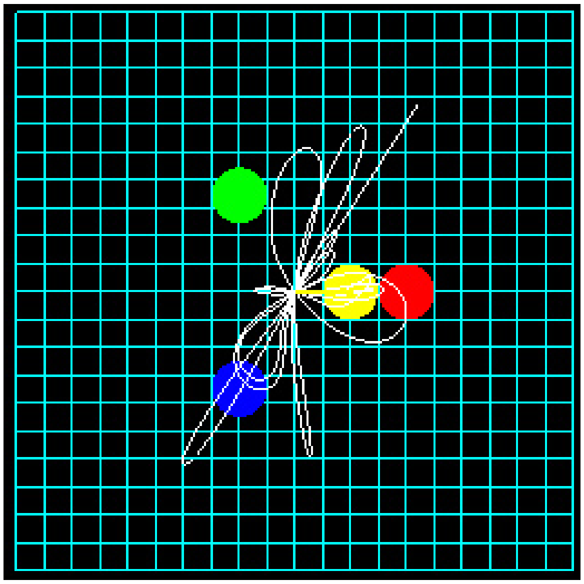
The C_i constants are each component's damping coefficients. There is some amount of air-resistance damping, but most of the damping is due to friction in the pivot point. These coefficients were obtained empirically, and are what cause the motion to eventually die out. The denominators are the moments of inertia for the two motions. The moments of inertia are different because the Φ motion uses the entire L while the Θ motion uses just the horizontal component of L.

The simulation used a second order Runge-Kutta scheme. This was a good compromise between accuracy and speed.

The Graphics Program

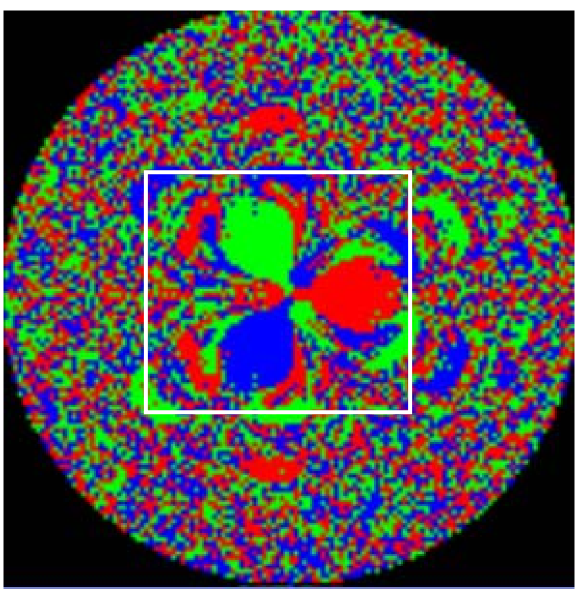
The graphics program was written using the OpenGL graphics API [Shreiner04] and the GL Utility Toolkit (GLUT) [GLUT05] window interface. The program presents three windows to the user: a 3D window showing the whole scene, a 2D window showing just the overhead view, and an image window showing the chaos pattern. The user can grab and move any of the magnets in the 2D window. The user can also use that same window to grab the pendulum bob, raise it to any position, and let it go. It then goes through its equations of motion, leaving a trail in both the 3D and 2D windows as shown below:

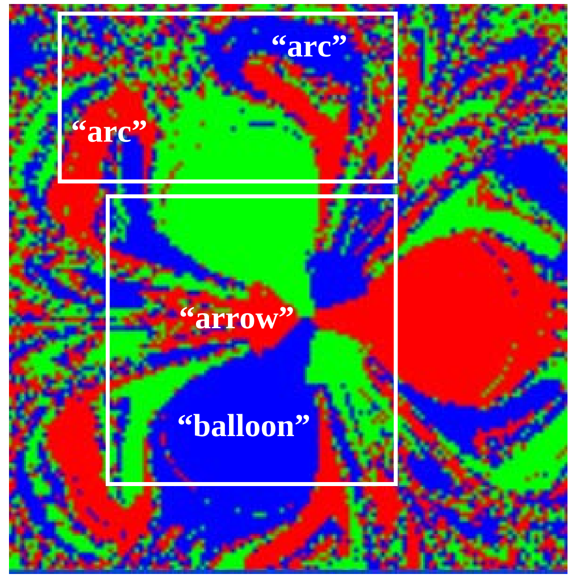


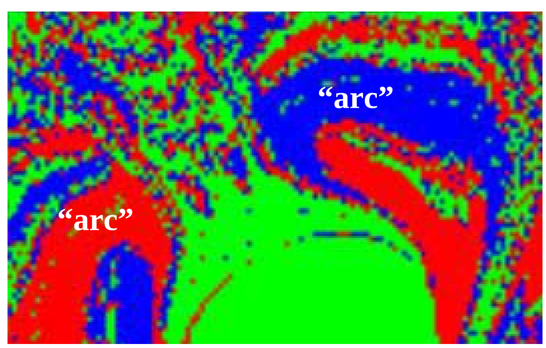


Results and Experimentation

For the magnet configuration shown above, the following three figures show the chaos pattern, progressively zoomed in. In this imaging pattern, each pixel represents the starting location for the pendulum bob, projected on the horizontal plane. It is in the same coordinate system as the 2D scene view shown above. The color at each pixel represents at which magnet the bob ultimately comes to rest.



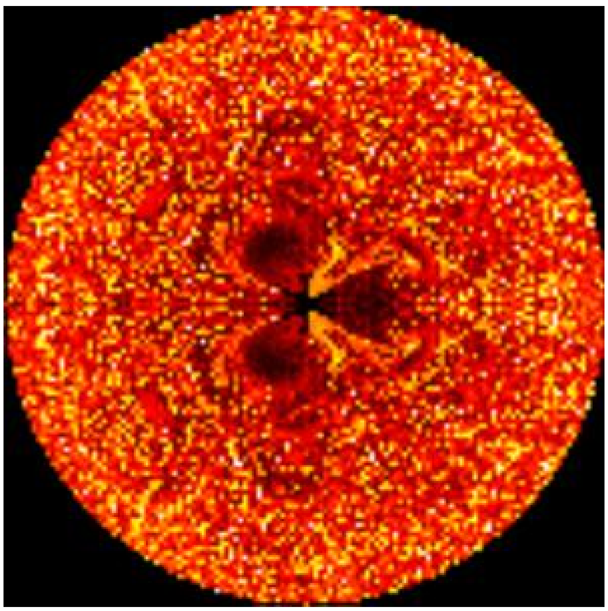




The patterns are fascinating. In the inner regions, there are several “balloons”, “arcs”, and “arrows”. The above images show zoomed-in areas. Like other chaotic patterns, such as the

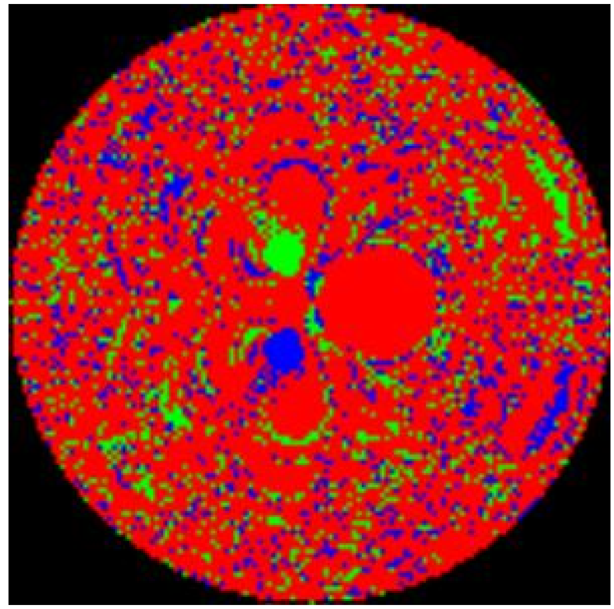
Mandelbrot set [Mandelbrot83], the zooming can go on forever. However, there do not appear to be any recurring multiresolution patterns like the "Mandelbrot ladybug".

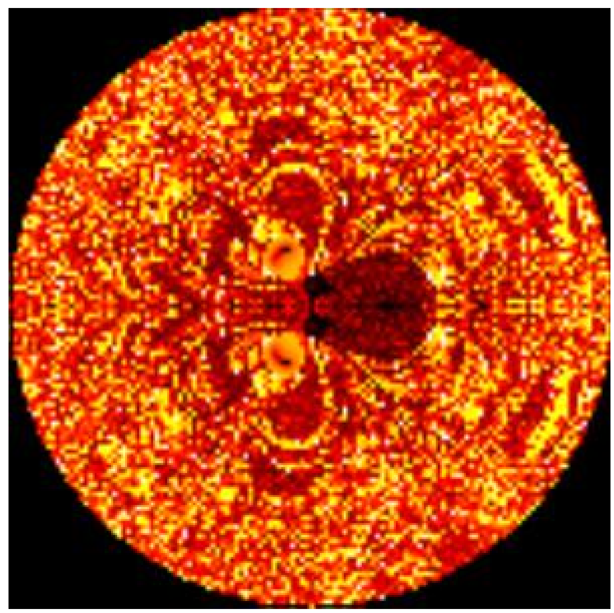
It is also interesting to see how long it takes the simulation to come to rest. The following image uses the same magnet configuration as the example above, and shows the relative number of simulation steps as a function of starting position. It is using a heated-object scale (black to red to yellow to white) as this uses both color and luminance to show relative value.



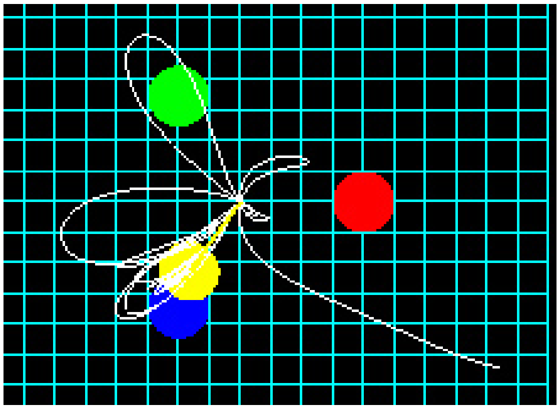
Intuitively, one would think that starting positions near the outer edge would converge more slowly because the bob is lifted higher and thus starts with more energy. However, this is not the case. The "balloons" from before capture the bob relatively quickly, but the convergence times for the rest of the starting positions is about as chaotic as the resting positions.

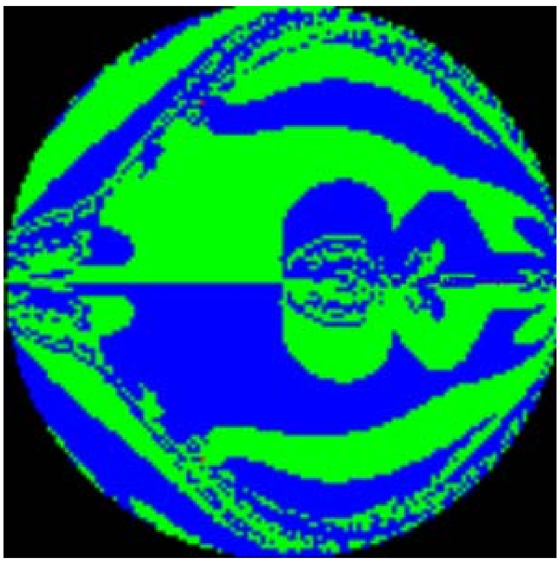
There are many ways to edit the scene, such as changing the position of the magnets, the strength of the magnets, the mass of the bob, gravity, and the damping. For example, the following two figures show the result of increasing the strength of the red magnet 25%. As can be seen, the bob comes to rest at the red magnet far more often than it did before, but, interestingly, there are still "strings of pearls" of green and blue amongst all the red.





As another example, the following images show what happens when the red magnet (with its original strength) is changed from attractive to repulsive. In the top figure, the path of the bob often approaches the red magnet, but then gets pushed away. The image of the resting locations is all green and blue as expected, and is anti-symmetric about the horizontal axis. There are large regions of solid color, but also a significant amount of scattering of one color inside the other's region.





Conclusions

A method for visualizing mechanical chaos has been developed. In this method, a single image shows the eventual stopping points for all initial conditions. It is then easy to compare the effect of different mechanical properties of the system such as pendulum length, coefficient of friction, and position and strength of the magnets.

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