

## A ADDITIONAL APPLICATION EXAMPLES

A steady open-channel flow, as shown in Fig. 13 (a), is disturbed by an submerged Gaussian-shaped sill on the bed: the flow is subcritical (Froude number  $\sim 0.1$ ) and turbulent (Reynolds number  $\sim 10^5$ ). The flow direction is from left to right, and the submerged sill spans across the channel. Fig. 13 (b) shows the mode surfaces that correspond to the eigenvalue space presented in Fig. 4. The disturbance caused by the sill is clearly visible from the large gap in all feature surfaces except the linear degenerate surface (green) and the real neutral surface (orange). This implies that the flow above the sill is shear-dominant and linear, which reflects the accelerating flow over the sill.

When zooming in on the sill (Fig. 14 (b)), we observe that the real neutral surface (orange) and the linear degenerate surface (green) are above and cover the linear balanced surface (blue) and the complex neutral surface (red). This indicates that the shear-dominant, accelerating flow is present above the rotation-dominant flow behind the sill due to flow separation. Such kinematic features are three-dimensional and complex. The complexity of the flow can be also detected by the triple degenerate curves in Fig. 14 (c). Recall that a triple degenerate curve represents the intersection of all mode surfaces. It is important to note that a triple degenerate curve cannot be terminated within the domain; the curve must be self-connected (ring formation) or terminate at the physical boundaries. In Fig. 14 (c), the triple degenerate curves are distributed sparsely above the sill, while the curves behind the sill are densely distributed and complex. Observing the triple degenerate curves allows us to identify the locations of complex (or orderly) flow regimes more easily, which can lead to additional research questions. The difference in the structures of feature surfaces shown in Fig. 14 (b) between upstream and downstream of the sill is prominent near the water surface.

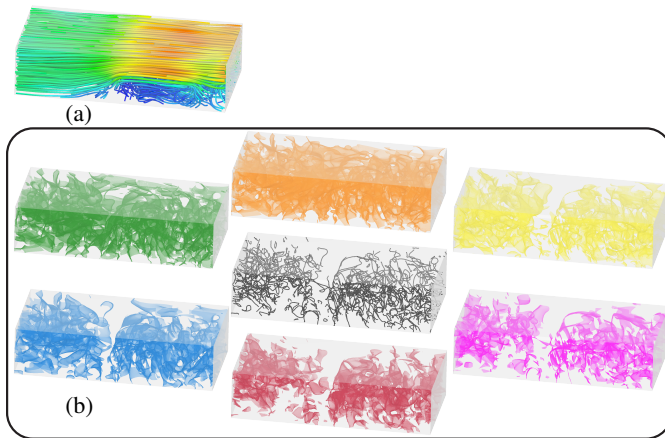


Fig. 13: This figure visualizes a steady open-channel flow with (a) streamlines and (b) six feature surfaces and the triple degenerate curve.

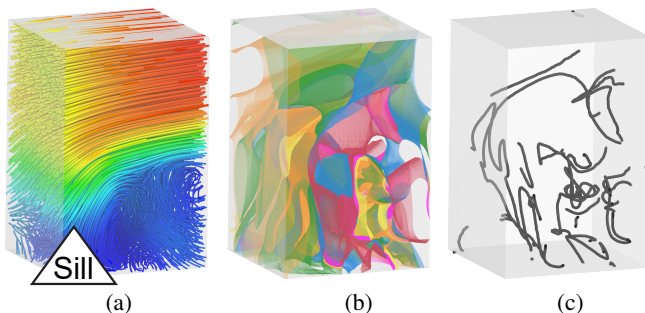


Fig. 14: This figure magnifies around the sill and show (a) the vector field, (b) the feature surfaces, and (c) the triple degenerate curve.